A Sequential Linear Optimization Approach for Controller Design

Lucas G. Horta* and Jer-Nan Juang†

NASA Langley Research Center, Hampton, Virginia

and

John L. Junkins‡

Texas A&M University, College Station, Texas

A linear optimization approach with a simple, real arithmetic algorithm is presented for reliable controller design and vibration suppression of flexible structures. Using first-order sensitivity of the system eigenvalues with respect to the design parameters in conjunction with a continuation procedure, the method converts a nonlinear optimization problem into a maximization problem with linear inequality constraints. The method of linear programming is then applied to solve the converted linear optimization problem. The general efficiency of the linear programming approach allows the method to handle structural optimization problems with a large number of inequality constraints on the design vector. The method is demonstrated using a truss beam, finite element model for the optimal sizing and placement of active/passive structural members for damping augmentation. Results using both the sequential linear optimization approach and nonlinear optimization are presented and compared. The insensitivity to the initial conditions of the linear optimization approach is also demonstrated.

Introduction

THE achievement of a reliable design for control and vibration suppression of large space structures is becoming more difficult due to stringent performance requirements such as pointing accuracy and other control maneuvers requiring very accurate controller design. This will likely be accomplished primarily by electromechanical devices attached to the structure, such that the structural behavior is modified and controlled efficiently during operation. The problem of determining proper placement and sizing of these devices must be solved in an iterative fashion. This iterative process is solved more efficiently if formulated as a nonlinear optimization problem.

The nonlinear optimization formulation gives rise to other complications. For example, the highly nonlinear or high-dimensional problems likely to occur in large space structures greatly reduce the convergence performance of the optimization algorithms. Thus, further analytic reduction of the original problem is desired to simplify the optimization task. Classical optimization techniques divide the problem into two phases: direction search and step length computations.[1-3] Although these techniques have proved to be very effective, a considerable reduction in the number of function and constraint evaluations can be obtained by combining these two steps.

This paper presents a linearized optimization approach that combines step length and direction search for controller design. The technique uses first-order sensitivity of the eigenvalues with respect to changes in the design parameters to obtain (in a single step) further improvements in the design vector. The sensitivity equation, which was derived in complex form in Ref. 4, has been transformed to the real domain using the complex eigenvectors as transformation basis.[5] To linearize the problem, a family of problems is defined by means of a continuation procedure as defined in Ref. 6. The continuation strategy begins with a known solution and then the next converged problem is defined and solved in a sequential manner until the final solution is reached. The procedure outlined in Ref. 6 used a minimum norm penalty to determine successive changes in the improvement vector for cases where the design variables exceed the number of binding constraints. In the proposed procedure, the improvement vector changes in each continuation step are determined by defining a maximization problem with linear inequality constraints; thus, linear programming is used.

The approach developed in this paper is implemented for the optimal sizing and placement of active/passive structural members (that exhibit variable stiffness or damping) in a truss beam. To evaluate this solution strategy, the problem was solved using both continuation method and standard nonlinear search algorithms. The insensitivity of the continuation procedure to various initial conditions is demonstrated.

Problem Statement

Consider a flexible structure such as the truss beam shown on Fig. 1, restrained at one end and free at the other end. The nth order discretized model can be written as

$$M \ddot{X} + C \dot{X} + K X = B U \qquad (1)$$

where M, K, and C are the ($n \times n$) mass, stiffness, and damping matrices, respectively, and X the state vector. The vector U represents applied control forces and B contains the locations and direction cosines of these applied forces.

Consider the case of linear output feedback control. For the case of position and velocity sensors, let Y_1 and Y_2 represent the measured position and velocity information

$$Y_1 = H_1 X, \qquad Y_2 = H_2 \dot{X} \qquad (2)$$

where Y_1 is an $m_1 \times 1$ vector, H_1 an $m_1 \times n$ matrix, Y_2 an $m_2 \times 1$ vector and H_2 an $m_2 \times n$ matrix. Constant gain matrices G_1 and G_2 are defined such that

$$U = -[G_1 Y_1 + G_2 Y_2] = -G_1 H_1 X - G_2 H_2 \dot{X} \qquad (3)$$

Substituting Eq. (3) into Eq. (1), the equation describing the closed-loop system is of the form

$$M \ddot{X} + \overline{C} \dot{X} + \overline{K} X = 0 \qquad (4)$$

*Aerospace Engineer, Structural Dynamics Branch. Member AIAA.

†Senior Research Scientist, Structural Dynamics Branch. Member AIAA.

‡Tees Professor of Aerospace Engineering. Associate Fellow AIAA.

where $\overline{C}$ and $\overline{K}$ are

$$\overline{C} = C + BG_2H_2, \quad \overline{K} = K + BG_1H_1$$

Let P represent a parameter vector defined as

$$P^T = \{p_1 \, p_2 \, \ldots \, p_{ndv}\} \tag{5}$$

where ndv corresponds to the number of design variables. Each of the matrices of Eq. (4) is a function of the system parameters P, including material and geometric properties, control devices location, control gains, etc., that can be expressed as

$$M = M(P), \quad \overline{K} = \overline{K}(P), \quad \overline{C} = \overline{C}(P) \tag{6}$$

The sensitivity of the system to changes in these parameters then can be determined as follows.

Rewriting Eq. (4) as a set of equivalent first-order differential equations yields

$$\overline{A} \, \dot{Z} = \overline{B} \, Z \tag{7}$$

for

$$Z = \begin{Bmatrix} X \\ \dot{X} \end{Bmatrix}, \quad \overline{A} = \begin{bmatrix} I & 0 \\ 0 & M(P) \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} 0 & I \\ -\overline{K}(P) & -\overline{C}(P) \end{bmatrix}$$

The sensitivity of the system eigenvalues to changes in the parameter vector P has been examined in Ref. 4 by considering the left and right eigenvalue problem (associated with the solution $Z = \phi e^{\lambda t}$), which is written as

$$\lambda_i \overline{A} \, \phi_i = \overline{B} \, \phi_i, \quad \lambda_i \overline{A}^T \, \psi_i = \overline{B}^T \, \psi_i \tag{8}$$

for $i = 1, 2 \ldots 2n$. The conventional normalization criterion is

$$\psi_j^T \overline{A} \, \phi_i = \delta_{ij}, \quad \psi_j^T \overline{B} \, \phi_i = \delta_{ij} \, \lambda_i \tag{9}$$

Differentiating Eq. (8) with respect to any element p_l of P, premultiplying the resulting equations by ψ_j^T and ϕ_j^T, and making use of the biorthogonality relations [Eq. (9)] thus yields

$$\frac{\partial \lambda_i}{\partial p_l} = \psi_i^T \left[\frac{\partial \overline{B}}{\partial p_l} - \lambda_i \frac{\partial \overline{A}}{\partial p_l} \right] \phi_i \tag{10}$$

Note that in this derivation all the eigenvalues are assumed distinct. When the eigenvalues are very close together, Eq. (10) may have to be replaced by the equation derived for repeated eigenvalues (see Ref. 4). Since the eigenvalues/eigenvectors are complex quantities, Eq. (10) requires complex arithmetic. An alternative would be to separate Eq.(10) into the real and imaginary parts and thus use real arithmetic. The following section describes another alternate and effective way to avoid the use of complex arithmetic.

Transformed Problem

The approach to be described herein takes advantage of the output format of most eigenvalue/eigenvector extraction techniques. Define $\lambda_\kappa = \mu_\kappa + i\beta_\kappa$ $(i = \sqrt{-1})$ $\kappa = 1, 2 \ldots 2n$ to be the set of system eigenvalues with corresponding eigenvectors $\phi_\kappa = \phi_\kappa^r + i\phi_\kappa^i$ or $\psi_\kappa = \psi_\kappa^r + i\psi_\kappa^i$. If all the eigenvectors are linearly independent, real transformation matrices using as basis the corresponding real and imaginary part of the eigenvectors, can be formed (see Ref. 5)

$$\Phi = [\phi_1^r \, \phi_1^i \, \phi_2^r \, \phi_2^i \, \ldots \, \phi_n^r \, \phi_n^i]$$

$$\Psi = [\psi_1^r \, \psi_1^i \, \psi_2^r \, \psi_2^i \, \ldots \, \psi_n^r \, \psi_n^i] \tag{11}$$

such that the eigenvalue problem of Eq. (8) becomes

$$\overline{A} \, \Phi \, \Lambda = \overline{B} \, \Phi, \quad \overline{A}^T \, \Psi \, \Lambda^T = \overline{B}^T \, \Psi \tag{12}$$

where the matrix Λ is a $2n \times 2n$ block-diagonal matrix composed
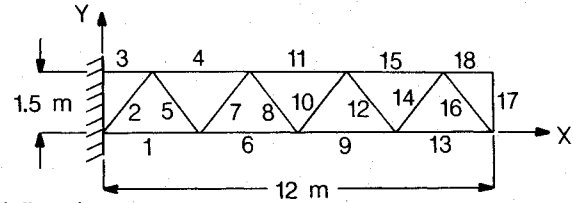
For hollow tubes
$\rho = 1700 \text{ kg/m}^2$
$E = 1.1 \times 10^{11} \text{ Pa}$
Longeron diameter = 3.75 cm
Diagonal diameter = 3.00 cm
Material thickness = 0.2 cm

Fig. 1   Truss beam finite elment model.

of 2×2 block matrices of the form

$$\Lambda_{ii} = \begin{bmatrix} \mu_i & \beta_i \\ -\beta_i & \mu_i \end{bmatrix} \tag{13}$$

Note that repeated eigenvalues do not violate the equalities of Eqs. (12) and (13), as long as all the corresponding eigenvectors are linearly independent. Recognizing that Φ and Ψ comprise a set of linearly independent basis vectors, premultiplying Eq. (12) by Ψ^T yields

$$\Lambda = \Psi^T \, \overline{B} \, \Phi \tag{14}$$

where the eigenvectors are normalized such that

$$\Psi^T \, \overline{A} \, \Phi = I \tag{15}$$

For the case where all the eigenvalues are assumed distinct, all the corresponding eigenvectors are linearly independent. Thus, the real equation equivalent to Eq. (10) can then be expressed as

$$S = \Psi^T \frac{\partial \overline{B}}{\partial p_l} \Phi - \Psi^T \frac{\partial \overline{A}}{\partial p_l} \Phi \, \Lambda \tag{16}$$

where S is a $2n \times 2n$ matrix. The sensitivity information is recovered by

$$2 \frac{\partial \mu_k}{\partial p_l} = S_{2k-1,2k-1} + S_{2k,2k}$$

$$2 \frac{\partial \beta_k}{\partial p_l} = -S_{2k,2k-1} + S_{2k-1,k}$$

$$k = 1, 2 \ldots n, \quad l = 1, 2 \ldots ndv \tag{17}$$

This set of algebraic equations completely avoids the use of complex arithmetic and simplifies the structure of the problem for further implementation. It should be recognized that, when the eigenvalues are very close together, Eqs. (10) and (17) may not be valid due to numerical difficulties. The sensitivity information [Eq. (17)] is directly applied in the optimization solution strategy as outlined in the next section.

Optimization Strategy

Consider the constrained optimization problem wherein an optimal solution vector P is sought such that a cost function is minimized,

$$\text{Minimize } F = F(\lambda_i(P), P) \tag{18}$$

subject to the inequality constraints

$$g_j(\lambda_i(P), P) \le 0, \quad j = 1, 2 \ldots ncon \tag{19}$$

where $ncon$ is the number of active constraints.

Numerous nonlinear direct and indirect optimization techniques have been developed and applied to the solution of these types of problems. Among the direct methods are the method of feasible directions,[1-3] gradient projection method, and minimum norm

procedure.[7] Classical optimization techniques divide the optimization strategy into two steps: direction search and step length computations. The direction search process usually is based on local linearization of the cost and constraint functions as

$$\Delta F = \left[\frac{\partial F}{\partial p_1} \frac{\partial F}{\partial p_2} \cdots \frac{\partial F}{\partial p_{ndv}} \right] \Delta P = J_f^T \Delta P \qquad (20)$$

and

$$\Delta g \le \left[\frac{\partial g}{\partial p_1} \frac{\partial g}{\partial p_2} \cdots \frac{\partial g}{\partial p_{ndv}} \right] \Delta P = J_g \Delta P \qquad (21)$$

with $\Delta P^T = [\Delta p_1 \ \Delta p_2 \ \ldots \ \Delta p_{ndv}]$, ΔF is the linearized expected change in the cost function, Δg is a $ncon \times 1$ vector of constraint changes, J_f^T a $1 \times ndv$ matrix of locally evaluated cost function gradients, J_g a $ncon \times ndv$ Jacobian matrix of locally evaluated constraints, and ΔP a vector of parameter changes.

Full advantage of these linearized equations can be taken if they are used not only for direction purposes but also in the step length determination of the parameter improvement vector ΔP. For example, the minimum norm procedure[7] is based on first-order per-turbation theory and relies on the Jacobian J_g and cost gradient J_f to obtain successive improvements in the estimated design vector while minimizing its norm. This is performed in a single-matrix operation. Nevertheless, a drawback of this technique is that, in order to solve explicitly for the improvement vector ΔP, the number of independent parameters ndv (design variables) must be greater or equal to the number of dependent parameters (active constraint equations). If this is not the case, the resulting system of equations has a unique or a least-square solution and no optimization is possible.

The advantage of the classical methods is their relative simplicity. On the other hand, their disadvantage is that they converge to a minimum of the cost function [see Eq. (18)], which is close to the initial value. If the initial value falls in the "domain of attraction"[8] for a particular local minimum, it will converge to the value of the parameter corresponding to the local minimum.

In contrast to these classical methods, a continuation method is used in this paper to solve the optimization problem for structure/controller design. Continuation methods have been fruitful over the last century in theoretical proofs of existence and uniqueness of various problems; see, for example, the historical survey of Ficken[9] and more recent works such as Refs. 10–12. However, the application of these methods to optimization problems for structure/controller design occurred only very recently.[6] The procedure to be developed herein does not use minimum norm penalty as in Ref. 6.

The continuation method starts with the construction of a new function $f_j(\alpha, P)$ usually referred to as the imbedded function

$$f_j(\alpha, P) = g_j(P) - (1 - \alpha)g_j(P_0) \le 0$$

$$j = 1, 2, \ldots ncon \qquad (22)$$

where, for simplicity, $g_j(P) = g_j[\lambda_i(P), P]$, α is a new parameter with the domain [0, 1], and the design parameter vector P is a function of the new parameter α. $P_0 = P(0)$ is the initial value of the parameter P whose value $g_j(P_0)$ is known but not satisfied by Eq. (19). Equation (22) has the following properties: 1) at $\alpha = 0$, this equation reduces to the base equation $f_j(0, P_0) = 0$ and 2) at $\alpha = 1$, it reduces to the original inequality constraint $f_j(1, P) = g_j(P) \le 0$. Therefore, as α varies continuously from 0 to 1, the parameter vector P varies continuously from $P(0)$ satisfying the constraint equation $f_j(0, P_0) = 0$ to $P(1)$ satisfying the constraint equation $f_j[1, P(1)] = g_j[P(1)] \le 0$.

Table 1 Initial trial vectors for cases 1–3

Location	Case 1	Case 2	Case 3
1	1.0	100.0	200.0
2	1.0	0.0	200.0
3	1.0	0.0	200.0
4	1.0	0.0	0.0
5	1.0	0.0	0.0
6	1.0	0.0	0.0
7	1.0	0.0	0.0
8	1.0	0.0	0.0
9	1.0	0.0	0.0
10	1.0	0.0	0.0
11	1.0	100.0	0.0
12	1.0	0.0	0.0
13	1.0	0.0	0.0
14	1.0	0.0	0.0
15	1.0	0.0	0.0
16	1.0	0.0	0.0
17	1.0	0.0	0.0
18	1.0	0.0	0.0

Table 2 Final design vector for different initial trial vectors

Location	Continuation approach, Cases 1–3	Nonlinear search Analytic gradient (ADS)[a] Case 1	Case 2	Case 3	Finite differences (CONMIN)[a] Case 1	Case 2	Case 3
1	525.7	421.8	451.9	450.1	340.2	620.0	1108.0
2	60.8	92.3	90.3	116.5	203.2	47.0	9.6
3	0.0	47.8	10.2	153.4	107.5	0.0	0.0
4	0.0	69.8	42.8	13.7	226.2	0.0	0.0
5	0.0	0.0	19.1	0.0	75.9	0.0	10.7
6	32.9	37.4	2.8	0.1	195.5	56.7	112.8
7	0.0	0.0	25.8	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	10.6	0.0	0.0
9	0.0	20.2	0.0	0.0	165.0	7.0	151.0
10	0.0	0.0	10.0	0.0	0.0	0.0	0.0
11	102.3	18.7	71.2	0.1	171.9	86.8	227.9
12	0.0	0.0	0.0	0.0	12.8	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	1.4	0.0	5.7	38.8	0.0	0.0
16	0.0	0.0	39.2	36.8	0.0	0.0	7.6
17	3.6	29.3	64.6	74.2	3.2	4.2	2.7
18	1.9	3.0	0.0	47.2	0.0	3.8	1.2
Final objective	727.2	741.7	827.9	897.8	1550.9	825.5	1631.5

[a]ADS and CONMIN are the program subroutines for nonlinear search available at NASA Langley Research Center.

The next step in the continuation method is to derive a differential equation that determines the trajectories of the parameter vector P, minimizing the function F as shown in Eq. (18). Differentiating Eq. (22) with respect to α yields

$$\frac{\partial g_j(P)}{\partial P} \frac{\partial P}{\partial \alpha} + g_j(P_0) = J_g \frac{\partial P}{\partial \alpha} + g_j(P_0) \le 0 \tag{23}$$

where $P = P_0$ when $\alpha = 0$. Equation (23) represents an initial value problem. Equations (18) and (23) thus represent an optimization problem involving the integration of Eq. (23) with respect to α and minimization of Eq. (18) with respect to $P(\alpha)$. The value $P(1)$ at $\alpha = 1$ becomes the desired parameter vector, which satisfies the constraint equation (19).

Note that Eq. (23) is only a sufficient condition that produces the inequality constraint Eq. (19) at the value $\alpha = 1$. There may exist another better imbedded function that releases the constraint condition in the open domain $0 < \alpha < 1$ and still reaches the final inequality equation (19) only when $\alpha = 1$. Existence of such imbedded functions require further investigation. The solution derived from Eqs. (18) and (23) may be conservative in the sense that the minimum obtained for Eq. (23) may not be the best for a given initial condition $P(0)$, because the solution trajectory may be overconstrained by Eq. (23) when $0 \le \alpha < 1$. However, it will be shown in the sample problem that this set of equations is insensitive to the initial conditions $P(0)$.

In contrast to classical integration methods such as Runge-Kutta, Newton-Raphson, finite differences, etc., a new approach involving standard linear programming is introduced to solve Eq. (23) and simultaneously minimize the cost function [Eq. (18)]. For small changes in α, say $\Delta\alpha$, Eqs. (18) and (23) can be written as the following auxiliary problem:

$$\text{Maximize } -\Delta F = -J_f^T \Delta P \tag{24}$$

subject to

$$J_g \Delta P \le -g_j(P_0) \Delta\alpha \tag{25}$$

where $g_j(P_0) \Delta\alpha$ are known desired changes in the local constraints. The solution to this auxiliary problem can be used to solve the nonlinear optimization problem of Eqs. (18) and (23). This is accomplished by restructuring the constrained optimization problem as a sequence of constraint satisfaction problems with small $\Delta\alpha$.

Looking at the "constraint only" version of the problem, the solution of the auxiliary problem seeks an improvement vector ΔP that locally maximizes the predicted improvement Eq. (24) in F

subject to Eq. (25) (first-order satisfaction of the constraints). The vector of constraint changes $g_j\Delta\alpha$ is set a priori and is based on its distance to the feasible region.

The problem presented by Eqs. (24) and (25) can be transformed further to a standard linear programming problem. Let $\Delta P = X_1^T - X_2^T$, where $X_1^T \ge 0$, $X_2^T \ge 0$, and $\bar{S}^T$ be a $ncon \times 1$ vector ($\bar{S}^T \ge 0$) of slack variables (S takes up the slack of the inequality constraints). Substituting these into Eqs. (24) and (25), the transformed linear programming problem posed is then

$$\text{Maximize } -\Delta F = -J_f^T\{X_1^T - X_2^T\} \tag{26}$$

subject to:

$$J_g(X_1^T - X_2^T) - \bar{S}^T = g_j(P_0)\Delta\alpha \tag{27}$$

where the total number of variables is $(2ndv + ncon)$. Note that, in contrast to the approach shown in Ref. 6, the existence of a solution to the linear programming problem is independent of the number of constraint or design variables. If a solution exists, the number of iterations to reach it is related linearly to the number of variables in the transformed problem. Observe that a zero entry in J_f for a linear objective function implies removing a design variable from the linear programming problem.

Once the solution to the auxiliary problem is obtained, successive iteration using $P + \Delta P$ will drive the cost function $F[\lambda_i(P), P]$ to a minimum and will move the constraint functions $g_j[\lambda_i(P), P]$ to a feasible region. In other words, for $0 < \alpha < 1$, a family of neighboring problems is defined wherein each previous solution bootstraps the solution of the next problem. The procedure allows the natural flow from a starting trial known solution P_0 to the desired solution $P(\alpha = 1)$. Since the flow is directly controlled by the number of continuation steps, the linearity assumption can be preserved.

Sample Problem

Consider the problem of damping augmentation[13,14] of a truss beam. Damping of the beam is to be increased by 1% in all modes. The geometry of a NASTRAN finite element (tube elements) model of a clamped truss beam shown on Fig. 1 (the numbers correspond to the member location) is used in this analysis to demonstrate the procedure. The structural model has nine nodes with two degrees of freedom per node. To solve this problem, two approaches are used: 1) viscous dampers can be placed properly along the beam members to achieve a certain damping ratio passively; or 2) control forces can be applied discretely in such a way that the closed-loop system eigenvalues are located properly. Although these two approaches are very different from the implementation viewpoint,

Table 3 Final constraint values (modal damping, %)

	Continuation approach, Cases 1–3	Nonlinear Search					
		Analytic gradient (ADS)			Finite differences (CONMIN)		
Mode no.		Case 1	Case 2	Case 3	Case 1	Case 2	Case 3
1	1.0	1.0	1.0	1.0	1.2	1.2	2.1
2	1.0	1.0	1.0	1.0	3.6	1.0	2.3
3	7.1	6.8	6.8	6.2	9.8	8.2	14.5
4	1.0	2.1	1.8	2.1	6.8	1.0	2.1
5	3.7	3.9	4.9	4.3	8.9	3.2	4.1
6	11.8	12.1	12.6	1.3	18.3	11.5	2.3
7	4.7	5.6	5.3	5.1	16.7	4.1	10.9
8	7.2	3.4	4.8	2.9	5.8	10.3	3.4
9	1.4	2.0	4.7	2.2	2.3	1.2	3.3
10	1.0	1.2	4.6	3.5	4.5	1.3	1.0
11	2.5	3.6	3.0	5.7	10.9	2.1	9.7
12	23.3	21.0	23.9	19.8	20.0	37.5	100.0
13	3.5	4.0	5.5	6.5	17.3	3.6	10.4
14	14.4	11.7	12.6	11.7	49.6	11.2	25.5
15	1.7	5.2	5.1	9.3	55.4	1.8	4.9
16	6.8	15.0	12.1	5.5	13.9	6.4	21.5
17	1.0	8.0	18.9	21.7	1.0	1.2	1.0
18	1.0	1.1	1.2	11.5	2.6	1.4	1.5

they are identical from the aspect of mathematical system design. This will now be addressed.

In the first approach for placement of member dampers, the matrix $[\overline{C}]$ in Eq. (4) can be written as

$$\overline{C} = \sum_{l=1}^{ndv} \overline{T}^T C_l \overline{T} \tag{28}$$

where $\overline{T}$ is a matrix transformation from local to global coordinates and C_l a 2×2 member damping matrix of the form,

$$[C_l] = \begin{bmatrix} c_l & -c_l \\ -c_l & c_l \end{bmatrix} \tag{29}$$

with c_l being the member dashpot coefficient. The subscript l also corresponds to the location along the truss beam shown on Fig. 1.

In the second approach, assuming axially collocated sensors and actuators, output velocity feedback, and zero initial system damping, the closed-loop matrix $\overline{C}$ [Eq. (4)] is expressed as

$$\overline{C} = B G_2 H_2 \tag{30}$$

where G_2 is diagonal and $H_2 = B^T$ contains the direction cosines of the corresponding sensor/actuator location. It can be shown that the two $[\overline{C}]$ expressions from Eqs. (28) and (30) are equivalent to

$$B G_2 H_2 = \sum_{i=1}^{ndv} \overline{T}^T C_i \overline{T} \tag{31}$$

if the diagonal elements of G_2 are constrained to have positive values to guarantee the system stability. Output velocity feedback with collocated sensors and actuators has been referred to as "modal dashpot" in Ref. 14. It is a generalization of the member damper concept.

Proceeding with the first approach (since they are the same), damping augmentation of a truss beam can be studied by posing the following problem:

$$\text{Minimize } F(c_i) = \sum_{i=1}^{ndv} c_i \tag{32}$$

subject to

$$\zeta_{obj} - \zeta_j \leq 0 \quad \text{for } j = 1, 2, \ldots ncon \tag{33}$$

where ζ_{obj} is the desired damping ratio and ζ_j the actual damping ratio.

Using the previous notation $P^T = \{c_1 c_2 \ldots c_{ndv}\}$ for $ndv = 18$ and $g_j(c_i) = \zeta_{obj} - \zeta_j$ for $j = 1, 2, \ldots ncon$ ($ncon = 18$), the gradient matrices can be evaluated using the chain rule of partial differentiation and Eq. (10) or (16). In this example, $\partial A/\partial p_i$ is a null matrix and $\partial B/\partial p_i = \overline{T}_i^T \overline{T}_i$. Since no optimal location among the truss beam members is known a priori, the optimization allows the damper placement at all locations. The damping ratio is constrained to be greater or equal to 1% for all the system modes.

The auxiliary problem presented in Eqs. (24) and (25) is augmented in order to constrain $P \geq 0$. The problem posed is

$$\text{Maximize } -\Delta F = -J_f^T \Delta P \tag{34}$$

subject to

$$J_g \Delta P \leq -g_j(P_0)\Delta \alpha \tag{35}$$

$$P + \Delta P \geq 0$$

It is important to notice that converged results are required for each suboptimization problem. To achieve this, several iterations per continuation step may be necessary.

To investigate the sensitivity of the continuation procedure to initial conditions and also compare the solution with standard non-linear programming techniques, the trial vectors P_0 (cases 1–3) shown in Table 1 are used. Table 2 shows the corresponding location along the truss beam (see Fig. 1) where dashpots are placed, followed by the solution vector for cases 1–3 from the continuation procedure (using five steps $\alpha = 0.2, 0.4, 0.6, 0.8,$ and 1.0). Note that many of the damper coefficients are zero, thus discarding that location for damper placement. Also shown in Table 2 are the results from nonlinear search (ADS[15] and CONMIN[16]) using the analytically derived gradients and internally calculated gradients (finite differences). The continuation procedure yields the same solution for each case, whereas the nonlinear search produces different solutions. For the cases shown, the smallest objective function was obtained using the continuation procedure. Fairly good results were obtained from nonlinear programming when the analytic gradients are provided, with some computational overhead. No attempt was made to optimize the input parameters of the nonlinear search routines because the primary objective is to demonstrate the sensitivity of these techniques to different initial conditions. Table 3 shows the final modal damping values in each of the 18 structural modes. High damping rates appear in some modes because of the localized nature of the mode.

Conclusions

An optimization approach using a continuation procedure in conjunction with linear programming has been developed. It was shown to be numerically more effective and less sensitive to initial conditions than classical optimization techniques and eliminates the arbitrary selection of initial conditions. For most large space structure and control applications, this method appears to be an extremely attractive alternative to other existing techniques, since large dimensions can be handled more efficiently by linear programming. Although the system eigenvalues are assumed distinct during the development of this method, the optimization strategy still applicable for the case with repeated eigenvalues.

References

[1]Zoutendijk, G., *Methods of Feasible Directions*, Elsevier Press, Amsterdam, 1960, Sec. 8.2–9.5.

[2]Vanderplaats, G. N. and Roses, F., "Structural Optimization by Methods of Feasible Directions," *Computer and Structures*, Vol. 3, 1973, pp. 739–755.

[3]Gwin, L. B. and Vanderplaats, G. N., "The Auxiliary Problem for Feasible Directions," *AIAA Journal*, Vol. 13, May 1975, pp. 695–697.

[4]Plaut, R. H. and Huseyin, K., "Derivatives of Eigenvalues and Eigenvectors in Non-Self-Adjoint System," *AIAA Journal*, Vol. 11, Feb. 1973, pp. 250–251.

[5]Gantmacher, F. R., *Theory of Matrices*, Vols. I and II, Chelsea Publishing Co., New York, 1959.

[6]Bodden, D. S. and Junkins, J. L., "Eigenvalue Optimization Algorithm for Structural/Controller Design Iterations," *Journal of Guidance, Control, and Dynamics*, Vol. 8, Nov.–Dec. 1985, pp. 697–706.

[7]Glassman, B. A. et al., "A Parameter Optimization Procedure for Multi-Stage Vehicles," Paper 66-111 presented at AAS Space Flight Mechanics Specialist Conference, Denver, CO, July 1966.

[8]Collatz, L., *Functional Analysis and Numerical Mathematics*, Academic Press, New York, 1966, Chap. 2.

[9]Ficken, F. A., "The Continuation Method for Functional Equations," *Communications on Pure and Applied Mathematics*, Vol. 4, 1951, pp. 435–456.

[10]Meyer, G. H., "On Solving Nonlinear Equations With a One-Parameter Operator Imbedding," *SIAM Journal of Numerical Analysis*, Vol. 5, 1968, pp. 739–752.

[11]Laasonen, P., "An Imbedding Method of Iteration with Global Convergence," *Computing*, Vol. 5, 1970, pp. 253–258.

[12]Ortega, J. M. and Rheinboldt, W. C., *Iterative Solution of Nonlinear Equations in Several Variables*, Academic Press, New York, 1970.

[13]Henderson, T. C. and Canavin, J. R., "Damping Augmentation for Large Space Structures," AIAA Paper 78-490, 1978, pp. 207–213.

[14]Canavin, J. R., "Control Technology for Large Space Structures," AIAA Paper No. 78-1691, Sept. 1978.

[15]Vanderplaats, G. N., "ADS—A FORTRAN Program for Automated Design Synthesis," NASA CR 172460, Ver. 1.00, May 1984.

[16]Vanderplaats, G. N., "CONMIN—A FORTRAN Program for Constrained Function Minimization User's Manual," NASA TM X-62,282, Aug. 1973.